RELAXATION OF A FLUIDIZED BED TO THE STATE OF A CONTINUOUS MEDIUM

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The kinetic equation proposed in [1, 2] for describing the behavior of a system of particles in a gas flow differs from the usual Boltzmann equation with respect to the additional terms that take into account random variations of the particle velocity under the influence of the flow. As shown in [2], the collision operator and the Brownian-type operator in the starting kinetic equation describe essentially different simultaneous physical processes of change of state of the particle system: equalization of the mean kinetic energy of the particles and change of energy due to the action of the viscous forces associated with the suspending flow. Therefore the method of solving the kinetic equation used in [2], a direct generalization of the Chapman-Enskog method of solving the kinetic equation it is necessary to investigate method of solving the kinetic equation it is necessary to investigate the relaxation processes in the system. Moreover, the relaxation of systems of the fluidized-bed type to the continuum state is also of independent interest in connection with the analysis of fast processes in the system, i.e., processes with a characteristic duration of the order of the mean free time.

§1. The starting kinetic equation [1, 2] has the form:

$$\frac{\partial f}{\partial t} + u_i \frac{\partial f}{\partial x_i} + \left[\Phi(n, |\mathbf{q}-\mathbf{w}|)(q_i - w_i) - \frac{\partial U}{\partial x_i}\right]\frac{\partial f}{\partial u_i} = C(ff) + \frac{\partial}{\partial u_i}\left[\Phi(n, |\mathbf{q}-\mathbf{w}|)(u_i - w_i) f + B_{ij}\frac{\partial f}{\partial u_j}\right],$$
$$B_{ij} = B(q_i - w_i)(q_j - w_j). \tag{1.1}$$

Here $f(t, x, u)$ is the single-particle distribution function normalized by the mean number of particles per unit volume, $C(ff)$ is the collision operator [3], U is the potential energy of the external mass forces, q_i are the components of the mean carrier flow velocity along the axes of the fixed Cartesian coordinate system x_i, and Φ is some known function.

Moreover, we have

$$n = \int f d\mathbf{u}, \quad w_i = \frac{1}{3n}\int u_i f d\mathbf{u},$$
$$B = D\Phi\left\{\frac{1}{\varepsilon} - \frac{\partial \ln \Phi}{\partial \varepsilon}\right\}^2 \langle(\Delta\varepsilon)^2\rangle,$$
$$\varepsilon = 1 - nv_0. \tag{1.2}$$

Here v_0 is the particle volume, D is a constant, and $\langle(\Delta\varepsilon)^2\rangle$ is the mean-square fluctuation of the relative volume occupied by the gas flow.

There is a natural relationship between Eq. (1.1), the transport equations

$$\frac{\partial n}{\partial t} + \frac{\partial n w_i}{\partial x_i} = 0,$$
$$mn\left(\frac{\partial w_i}{\partial t} + w_\alpha \frac{\partial w_i}{\partial x_\alpha}\right) = \frac{\partial P_{i\alpha}}{\partial x_\alpha} + mn\left[\Phi(q_i - w_i) - \frac{\partial U}{\partial x_i}\right],$$
$$\frac{\partial \theta}{\partial t} + w_\alpha \frac{\partial \theta}{\partial x_\alpha} = \frac{2}{3n}\left[\frac{\partial Q_\alpha}{\partial x_\alpha} + P_{\alpha\beta}\frac{\partial w_\alpha}{\partial x_\beta}\right] + \frac{2}{3}mB|q-w|^2 - 2\Phi\theta,$$
$$\theta = \frac{m}{3n}\int|\mathbf{u}-\mathbf{w}|^2 f d\mathbf{u}, \tag{1.3}$$

where m is the particle mass, and the equations for determining the dynamic behavior of the gas flow

$$\rho_0(1 - nv_0)\left(\frac{\partial q_i}{\partial t} + q_\alpha \frac{\partial q_i}{\partial x_\alpha}\right) = -\frac{\partial \Pi}{\partial x_i} - \rho_0(1-nv_0)\frac{\partial U}{\partial x_i} - mn\Phi(q_i - w_i),$$
$$\frac{\partial}{\partial x_i}[nv_0 w_i + (1 - nv_0) q_i] = 0. \tag{1.4}$$

Here Π is the gas-flow pressure and ρ_0 is its density.

In (1.1), (1.3), and (1.4), as everywhere in what follows, the summation convention applies (from 1–3).

The quantities Q_α and $P_{\alpha\beta}$ in (1.3) are determined in the usual way [3].

§2. We denote by L the characteristic macroscopic scale of the system in question, by λ the mean distance between particles, and by w and c the characteristic macroscopic velocity of the mixture and the mean random velocity of the particles, respectively.

As is easily seen, Eqs. (1.1), (1.3), and (1.4) permit the introduction of the following characteristic time scales: a) mean free time between two successive collisions $\tau_1 = \lambda/c$; b) mean particle viscous drag time $\tau_2 = 1/\Phi$; c) convective time $\tau_3 = L/w$; d) characteristic spatial nonuniformity diffusional smoothing time $\tau_4 = L^2/\lambda c$.

If we denote by l the characteristic scale of viscous drag $l = c/\Phi$ and introduce the dimensionless parameters

$$\alpha = \lambda / L, \qquad \beta = l / L, \qquad M = w / c, \tag{2.1}$$

then for the ratios of the above-mentioned time scales we have

$$\tau_2 / \tau_1 = \beta / \alpha, \qquad \tau_3 / \tau_1 = 1 / M\alpha,$$
$$\tau_4 / \tau_1 = 1/\alpha^2. \tag{2.2}$$

When $M \sim 1$ and $\alpha \ll 1$ we always have $\tau_1 \ll \tau_3 \ll \tau_4$. The ratio τ_2/τ_1 may vary. We note that $\tau_2/\tau_1 = l/\lambda$. When the suspending flow has low viscosity, we have $\tau_1 \ll \tau_2$. Moreover, if $\tau_4 \ll \tau_2$, the system behaves like an ordinary gas and the suspending flow affects only the smoothing of the spatial inhomogeneities of the system. Therefore the most interesting case is that in which $\beta \sim 1$ and τ_2 is comparable in magnitude with τ_3. Under these conditions the gas flow will have an important influence on the dynamic behavior of the system, since the dimensions of the system are of the same order as the scale of viscous drag.

In accordance with the method proposed in [4], we seek the solution of (1.1), (1.3), and (1.4) in the form of a series in powers of a small parameter

$$f = f^{(0)} + \alpha f^{(1)} + \alpha^2 f^{(2)} + \ldots, \tag{2.3}$$

the functions $f^{(i)}$ depending on the variables

$$f^{(i)} = f^{(i)}(t_0, t_1, t_2, \ldots; \mathbf{u}, \alpha^{-1}\mathbf{x}), \tag{2.4}$$

where

$$t_0 = t, \quad t_1 = \alpha t, \quad t_2 = \alpha^2 t, \ldots. \tag{2.5}$$

Then the derivative of f with respect to time is written in the form

$$\frac{\partial f}{\partial t} = \frac{\partial f^{(0)}}{\partial t_0} + \alpha\left(\frac{\partial f^{(0)}}{\partial t_1} + \frac{\partial f^{(1)}}{\partial t_0}\right) + \alpha^2\left(\frac{\partial f^{(0)}}{\partial t_2} + \frac{\partial f^{(1)}}{\partial t_1} + \frac{\partial f^{(2)}}{\partial t_0}\right) + \ldots. \tag{2.6}$$

We also assume that the spatial nonuniformity of the system is relatively small and that the spatial derivatives in (1.1), (1.3), and (1.4) are of the order of α. From the assumption $\tau_2 \sim \tau_3$ it follows that

$$\Phi = \alpha A. \tag{2.7}$$

The quantities w_i, n, θ, A and $B_{\alpha\beta}$, will be certain functionals of f; therefore for each of these quantities there will be an expansion analogous to (2.3):

$$\begin{gathered} w_i = w_i^{(0)} + \alpha w_i^{(1)} + \alpha^2 w_i^{(2)} + \ldots, \\ n = n^{(0)} + \alpha n^{(1)} + \alpha^2 n^{(2)} + \ldots, \\ \theta = \theta^{(0)} + \alpha\theta^{(1)} + \alpha^2\theta^{(2)} + \ldots, \\ \alpha A = \alpha A^{(0)} + \alpha^2 A^{(1)} + \ldots, \\ B = \alpha B^{(0)} + \alpha^2 B^{(1)} + \ldots, \\ B_{\alpha\beta} = \alpha B_{\alpha\beta}^{(0)} + \alpha^2 B_{\alpha\beta}^{(1)} + \ldots. \end{gathered} \tag{2.8}$$

§3. Substituting (2.3) and (2.6) into (1.1), (1.3), as the zero-order approximation we have

$$\begin{gathered} \partial f^{(0)}/\partial t_0 = C(f^{(0)}f^{(0)}), \\ \partial w_i^{(0)}/\partial t_0 = \partial n^{(0)}/\partial t_0 = \partial\theta^{(0)}/\partial t_0 = 0\,. \end{gathered} \tag{3.1}$$

The expansion of Eq. (1.4) will not be a part of the zero-order approximation, if we note that for the case of a gas carrier, as follows from the known expressions [5, 6] for the function Φ, we have $\rho_0/\rho_d \sim \Phi \sim \alpha_1$, where ρ_d is the density of the solid.

From (3.1) it quickly follows that as $t_0 \to \infty$ the function $f^{(0)} \to f_M$, the Maxwellian distribution function. In order for f_M to correspond to the initial distribution in the Chapman-Enskog method, we require that $w_i^{(0)} \to w_i$, $n^{(0)} \to n$, $\theta^{(0)} \to \theta$ as $t_0 \to \infty$. Moreover, it is obvious that

$$P_{\alpha\beta}^{(0)} \to -p\delta_{\alpha\beta}, \quad Q_\alpha^{(0)} \to 0 \quad \text{as } t_0 \to \infty\,. \tag{3.2}$$

The equations of the first approximation are written in the form

$$\frac{\partial f^{(1)}}{\partial t_0} + \frac{\partial f^{(0)}}{\partial t_1} + u_i\frac{\partial f^{(0)}}{\partial x_i} + \left[A^{(0)}(q_i^{(0)} - w_i^{(0)}) - \frac{\partial U}{\partial x_i}\right]\frac{\partial f^{(0)}}{\partial u_i} = \frac{\partial}{\partial u_i}\left[A^{(0)}(u_i - w_i^{(0)})f^{(0)} + B_{ij}^{(0)}\frac{\partial f^{(0)}}{\partial u_j}\right] + C(f^{(0)}f^{(1)}) + C(f^{(1)}f^{(0)}), \tag{3.3}$$

$$\frac{\partial n^{(1)}}{\partial t_0} + \frac{\partial n^{(0)}}{\partial t_1} + \frac{\partial n^{(0)}w_i^{(0)}}{\partial x_i} = 0, \tag{3.4}$$

$$n^{(0)}m\left(\frac{\partial w_i^{(0)}}{\partial t_1} + \frac{\partial w_i^{(1)}}{\partial t_0} + w_\alpha^{(0)}\frac{\partial w_i^{(0)}}{\partial x_\alpha}\right) = \frac{\partial P_{i\alpha}^{(0)}}{\partial x_\alpha} + \left[A^{(0)}(q_i^{(0)} - w_i^{(0)}) - \frac{\partial U}{\partial x_i}\right]mn^{(0)}, \tag{3.5}$$

$$\frac{\partial\theta^{(0)}}{\partial t_1} + \frac{\partial\theta^{(1)}}{\partial t_0} + w_\alpha^{(0)}\frac{\partial\theta^{(0)}}{\partial x_\alpha} = \frac{2}{3n^{(0)}}\left[\frac{\partial Q_i^{(0)}}{\partial x_i} + P_{ij}^{(0)}\frac{\partial w_i^{(0)}}{\partial x_j}\right] + \frac{2}{3}mB^{(0)}|\mathbf{q}^{(0)} - \mathbf{w}^{(0)}|^2 - 2A^{(0)}\theta^{(0)}. \tag{3.6}$$

In exactly the same way for the carrier flow, we have

$$\frac{\partial}{\partial x_i}[n^{(0)}v_0 w_i^{(0)} + (1 - n^{(0)}v_0)q_i^{(0)}] = 0\,,$$

$$\frac{\rho_0(1 - n^{(0)}v_0)}{mn^{(0)}\alpha}\frac{\partial q_i^{(0)}}{\partial t_0} = -\frac{1}{mn^{(0)}}\frac{\partial\Pi^{(0)}}{\partial x_i} - A^{(0)}(q_i^{(0)} - w_i^{(0)}). \tag{3.7}$$

From the first of relations (3.7) it follows that $\partial q_i^{(0)}/\partial t_0 = 0$. In order to eliminate the secular terms in (3.4), we require that

$$\partial n^{(1)}/\partial t_0 = 0\,. \tag{3.8}$$

Then from the condition that $n^{(0)} \to n$ as $t_0 \to \infty$, we obtain $n^{(1)} = 0$. Similarly, eliminating the secular terms from (3.5) and (3.6), we have

$$mn^{(0)}\frac{\partial w_i^{(1)}}{\partial t_0} = \frac{\partial P_{ij}^{(0)}}{\partial x_j} + \frac{\partial p}{\partial x_i}\,,$$

$$\frac{\partial\theta^{(1)}}{\partial t_0} = \frac{2}{3n^{(0)}}\left[\frac{\partial Q_i^{(0)}}{\partial x_i} + (P_{ij}^{(0)} + p\delta_{ij})\frac{\partial w_i^{(0)}}{\partial x_j}\right]. \tag{3.9}$$

Equations (3.2) determine the transitional behavior of $w_i^{(1)}$ and $\theta^{(1)}$ if the transitional behavior of $f^{(0)}$ is known. And, finally, the asymptotic behavior of the function $f^{(1)}$ is determined by the equation

$$\frac{\partial f_M}{\partial t_1} + u_\alpha\frac{\partial f_M}{\partial x_\alpha} + \left[A^{(0)}(q_i^{(0)} - w_i^{(0)}) - \frac{\partial U}{\partial x_i}\right]\frac{\partial f_M}{\partial u_i} = \frac{\partial}{\partial u_i}\left[A^{(0)}(u_i - w_i^{(0)})f_M + B_{ij}^{(0)}\frac{\partial f_M}{\partial u_j}\right] + C(f_M f^{(1)}) + C(f^{(1)}f_M)\,, \tag{3.10}$$

which coincides with the second approximation in the Chapman-Enskog method. The asymptotic values of $Q_i^{(0)}$ and $P_{ij}^{(1)}$ give the expressions obtained in [2]. Continuing the expansion, we can obtain the next terms of the series and the equations for determining the transitional behavior of $Q_i^{(1)}$ and $P_{ij}^{(1)}$. The equations of the next approximation have the form

$$\frac{\partial f^{(2)}}{\partial t_0}+\frac{\partial f^{(1)}}{\partial t_1}+\frac{\partial f^{(0)}}{\partial t_2}+u_i\frac{\partial f^{(1)}}{\partial x_i}+$$

$$+[A^{(1)}(q_i^{(0)}-w_i^{(0)})+A^{(0)}(q_i^{(1)}-w_i^{(1)})]\frac{\partial f^{(0)}}{\partial u_i}+$$

$$+\left[A^{(0)}(q_i^{(0)}-w_i^{(0)})-\frac{\partial U}{\partial x_i}\right]\frac{\partial f^{(1)}}{\partial u_i}=C(f^{(2)}f^{(0)})+$$

$$+C(f^{(1)}f^{(1)})+C(f^{(0)}f^{(2)})+\frac{\partial}{\partial u_i}\Big[A^{(0)}(u_i-w_i^{(0)})f^{(1)}+$$

$$+A^{(1)}(u_i-w_i^{(0)})f^{(0)}-A^{(0)}w_i^{(1)}f^{(0)}+$$

$$+B_{ij}^{(0)}\frac{\partial f^{(1)}}{\partial u_j}+B_{ij}^{(1)}\frac{\partial f^{(0)}}{\partial u_j}\Big], \qquad (3.11)$$

$$\frac{\partial n^{(2)}}{\partial t_0}+\frac{\partial n^{(0)}}{\partial t_2}+\frac{\partial n^{(0)}w_i^{(1)}}{\partial x_i}=0, \qquad (3.12)$$

$$n^{(0)}m\left(\frac{\partial w_i^{(0)}}{\partial t_2}+\frac{\partial w_i^{(1)}}{\partial t_1}+\frac{\partial w_i^{(2)}}{\partial t_0}+\right.$$

$$\left.+w_\alpha^{(1)}\frac{\partial w_i^{(0)}}{\partial x_\alpha}+w_\alpha^{(0)}\frac{\partial w_i^{(1)}}{\partial x_\alpha}\right)=\frac{\partial P_{ij}^{(1)}}{\partial x_j}+$$

$$+mn^{(0)}[A^{(0)}(q_i^{(1)}-w_i^{(1)})+A^{(1)}(q_i^{(0)}-w_i^{(0)})], \qquad (3.13)$$

$$\frac{\partial\theta^{(0)}}{\partial t_2}+\frac{\partial\theta^{(1)}}{\partial t_1}+\frac{\partial\theta^{(2)}}{\partial t_0}+w_\alpha^{(0)}\frac{\partial\theta^{(1)}}{\partial x_\alpha}+w_\alpha^{(1)}\frac{\partial\theta^{(0)}}{\partial x_\alpha}=$$

$$=\frac{2}{3n^{(0)}}\left[\frac{\partial Q_i^{(1)}}{\partial x_i}+P_{\alpha\beta}^{(1)}\frac{\partial w_\alpha^{(0)}}{\partial x_\beta}+P_{\alpha\beta}^{(0)}\frac{\partial w_\alpha^{(1)}}{\partial x_\beta}\right]+$$

$$+\frac{2}{3}mB^{(1)}|\mathbf{q}^{(0)}-\mathbf{w}^{(0)}|^2+$$

$$+\frac{4}{3}mB^{(0)}(q_i^{(0)}-w_i^{(0)})(q_i^{(1)}-w_i^{(1)})-$$

$$-2A^{(1)}\theta^{(0)}-2A^{(0)}\theta^{(1)}, \qquad (3.14)$$

$$\frac{\rho_0(1-n^{(0)}v_0)}{mn^{(0)}\alpha}\left(\frac{\partial q_i^{(0)}}{\partial t_1}+q_\alpha^{(0)}\frac{\partial q_i^{(0)}}{\partial x_\alpha}+\frac{\partial q_i^{(1)}}{\partial t_0}\right)=$$

$$=-\frac{1}{mn^{(0)}}\frac{\partial\Pi^{(1)}}{\partial x_i}-\frac{\rho_0(1-n^{(0)}v_0)}{mn^{(0)}}\frac{\partial U}{\partial x_i}-$$

$$-A^{(1)}(q_i^{(0)}-w_i^{(0)})-A^{(0)}(q_i^{(1)}-w_i^{(1)}),$$

$$\frac{\partial}{\partial x_i}[n^{(0)}v_0w_i^{(1)}+(1-n^{(0)}v_0)q_i^{(1)}]=0. \qquad (3.15)$$

Eliminating the secular terms in (3.12)–(3.14), as $t_0\to\infty$ we obtain

$$\frac{\partial n^{(0)}}{\partial t_2}=0,\quad n^{(0)}m\frac{\partial w_i^{(0)}}{\partial t_2}=\frac{\partial P_{i\alpha}^{(1)}}{\partial x_\alpha},$$

$$\frac{\partial\theta^{(0)}}{\partial t_2}=\frac{2}{3n^{(0)}}\left[\frac{\partial Q_i^{(1)}}{\partial x_i}+P_{ij}^{(1)}\frac{\partial w_i^{(0)}}{\partial x_j}\right], \qquad (3.16)$$

if, as before, we require that $q_i^{(1)}\to 0$ as $t_0\to\infty$.

§4. The qualitative characteristics of the relaxation processes in the system can be studied for a relaxation model of the collision integral in exactly the same way as in [4]. Only the effects associated with the additional terms in equation (1.1) are of special interest, and in order to use the results of [4] we denote by f_* the solution of (1.1) at $A\equiv 0$ and investigate the behavior of the function $f-f_*$. Adding (3.16) and the expressions that follow from (3.4)–(3.6) after elimination of the secular terms, we find the equations of hydrodynamics of the system previously obtained in [2].

After elimination of the secular terms as $t_0\to\infty$ Eq. (3.11) leads to the third-approximation equation in the Chapman-Enskog method for starting equation (1.1).

Thus, the procedure used to solve the kinetic equation (1.8) in [2] leads to the asymptotic solution of that equation as $t_0\to\infty$.

For the relaxation model of the collision integral in the simplest case, following [4], we assume

$$C(ff)=\frac{f_M-f}{\tau(n,\theta)}. \qquad (4.1)$$

Using the above-mentioned expansion procedure for $f=f_*$, we find $f=f_*$.

Thus, the relaxation processes for Eq. (1.1) in the zero-order approximation coincide with the relaxation processes in the ordinary gas.

As may be seen from (3.9) and the corresponding expansion for $f_*^{(1)}$, the transitional behavior of $w_i^{(1)}$, $n^{(1)}$, and $\theta^{(1)}$ will be the same as that of $w_{i*}^{(1)}$, $\theta_*^{(1)}$, $n_*^{(1)}$.

We denote by φ the difference $f^{(1)}-f_*^{(1)}$. Then, for φ we have

$$\frac{\partial\varphi}{\partial t_0}+\frac{\varphi}{\tau_0}=A^{(0)}(w_i^{(0)}-q_i^{(0)})f^{(0)}+$$

$$+\frac{\partial}{\partial u_i}\left[A^{(0)}(u_i-w_i^{(0)})f^{(0)}+B_{ij}^{(0)}\frac{\partial f^{(0)}}{\partial u_j}\right],$$

$$\tau_0=\tau(n^{(0)},\theta^{(0)}),$$

$$f^{(0)}(t_0,t_1,\ldots;\mathbf{u},\alpha^{-1}\mathbf{x})=f_M(t_1,\ldots;\mathbf{u},\alpha^{-1}\mathbf{x})+$$

$$+[f^{(0)}(0,t_1,\ldots;\mathbf{u},\alpha^{-1}\mathbf{x})-f_M(t_1,\ldots;\mathbf{u},\alpha^{-1}\mathbf{x})]e^{-t_0/\tau_0}. \qquad (4.2)$$

Now let L be an operator acting on the function $f^{(0)}$ on the right side of (4.2). The solution for φ can now be written in the form

$$\varphi(t_0,t_1,\ldots;\mathbf{u},\alpha^{-1}\mathbf{x})=$$

$$=t_0e^{-t_0/\tau_0}L(f_0^{(0)}-f_M)-\tau_0(1-e^{-t_0/\tau_0})Lf_M,$$

$$f_0^{(0)}=f^{(0)}(0,t_1,\ldots;\mathbf{u},\alpha^{-1}\mathbf{x}). \qquad (4.3)$$

As may be seen from (4.3), the relaxational behavior of φ is characterized by the number of the exponential approximation of φ to its asymptotic value.

Using φ and the results of [4], we can determine the transitional behavior of $Q_\alpha^{(1)}$ and $P_{\alpha\beta}^{(1)}$. Then structure of the corresponding expressions is rather clumsy and accordingly they have been omitted.

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